Chapter R  Review of Basic Algebra

The goal of this chapter is to make the transition from arithmetic to algebra, from working with numbers to working with letters that represent numbers. Working means performing the basic operations of addition, subtraction, multiplication, and division, as well as extraction of roots.

The transition also requires familiarity with a larger number system. The numbers we deal with in everyday arithmetic consist mostly of positive integers 1, 2, 3, …, called the natural numbers, and fractions, called rational numbers. But in algebra, we also deal with negative numbers, 0, irrational numbers, such as and, and even complex numbers, such as, which we will meet in Chapter 2. Our goal here is to become familiar with the real number system.

Transitioning from specific numbers to letters representing numbers is an abstraction that requires deeper thinking. Rules that we take for granted in arithmetic, such as or become and, which are called the commutative rule and associative rule for addition, respectively. Our goal here is to become familiar with, and name, the many rules that govern algebra and with manipulating basic algebraic expressions.
In Chapter R we saw how the properties of addition, subtraction, multiplication, division, and extraction of roots for real numbers can be applied to basic algebraic expressions. The goal of this chapter is to apply these properties to more general algebraic expressions. Specifically, we consider polynomials, such as $3x^2 + x - 2$, rational expressions, such as $(x^3 - 3x^2 + 2x - 1)/(x - 2)$, expressions with rational exponents, such as $x^{3/2}$, and radicals, such as $\sqrt{x^2 - 4x + 5}$. 
Chapter 2  Algebra and Graphs of Linear Expressions

In the previous two chapters we were concerned with manipulating algebraic expressions. We refer to that activity as static algebra. In this chapter we embark on dynamic algebra. Our goal is to explore all aspects of linear expressions, that is, expressions of the form $ax + by + c$ or $ax + by + cz + d$. Specifically, our goal is to solve linear equations and inequalities as well as applications of them, to see how linear equations correspond to lines in the plane or planes in space, and to solve systems of linear equations.
In the previous chapter we explored all aspects of linear expressions. Our goal here is to do the same for quadratic expressions, that is, expressions of the form \( ax^2 + bx + c \) or \( ax^2 + by^2 + cx + dy + e \). Specifically, our goal is to solve quadratic equations and inequalities as well as applications of them, and to see how quadratic equations correspond to circles, parabolas, ellipses, and hyperbolas in the plane.
Chapter 4  Functions and Their Graphs

In this Chapter we take a step up in the level of mathematical abstraction and encounter the notion of a function, a concept that dominates almost every branch of mathematics. Our goal is to become familiar with the definitions of, and operations with, polynomial functions, such as \( f(x) = 2x^2 - 3x + 4 \), rational functions, such as \( \frac{x^3 - 3x^2 + 5}{x^2 - 1} \), and general algebraic functions, such as \( \sqrt{x^2 - 1} / (x^2 + 1) \), as well as the concepts of one-to-one and inverse functions.
Chapter 5  Exponential and Logarithmic Functions

The functions studied in the previous chapter are called algebraic functions because they can be defined in terms of addition, subtraction, multiplication, division, and extraction of roots. In this chapter we study the exponential function \( f(x) = a^x \) and the logarithmic function \( g(x) = \log_a x \), which are called transcendental functions because their definitions require powers with irrational exponents, such as \( 2^{\sqrt{5}} \), which is a non-algebraic operation. Our goal is to become familiar with the properties, graphs, and applications the exponential and logarithmic functions.
Chapter 6 Additional Topics

The content of the six previous chapters comprise all of the algebraic topics, as well as the exponential and logarithmic functions, needed to proceed to calculus. Our goal here is become familiar with a variety of topics that can be used to enhance any course in college algebra. Specifically, we wish to become familiar with some of the following topics: the binomial theorem, linear programming, partial fractions, Gaussian elimination, matrix algebra, and the theory of equations. Although it is unlikely that all of these topics can be covered in a one-semester course in college algebra, instructors can choose topics suitable for their class, and students can choose topics to read on their own.
Chapter Assessment

The Review Exercises at the end of each chapter serve as a thorough student self-assessment for the chapter material. After working through the self-assessment, students can check their answers in the Answers-to-Selected-Exercises section at the end of the textbook. This section contains the solutions to all of the review exercises.